Truss Modeler

Joseph Li, Christopher Kotelnick, and Angel Ordonez-Retamar

12/6/2021

Stevens Institute of Technology

Course E-231: Structural Performance and Failure

Group 4, Section C

We pledge on our Honor that we have abided by the Stevens Honor System.

Abstract:

Trusses are common building structures often used in bridges and rooftops. In this project, we were tasked with creating three different designs for a bridge whose load bearing structure was composed of a truss. We would then determine the loads at which the bridge would fail, and identify the weakest members in the structure. This report details the three group member's truss designs, the results from the truss analyzer, and the group's discoveries throughout the process. The group concluded that simpler designs were best, and that smaller members generally meant stronger members. The target of the report is an individual with knowledge of engineering concepts and trusses, but individuals with no prior knowledge should be able to learn quite a bit of new information from this paper.

Table of Contents:

<u>Introduction</u>

The Truss is a staple design in architecture, used in roof supports, and in the case of this project, bridges. The purpose of this project is to develop a modeler for trusses to determine failure loads. The modeler is to be generally applicable to all trusses, with the ability for the user to enter their own truss data to determine the failure load of their truss. Specifically, by inputting the output lengths and forces on each member of a truss under a given load from the Truss Analyzer developed by Roger Kleinman into our modeler, the modeler is able to determine the load at which members of the truss will fail, both theoretically and empirically. First, the modeler must take into account certain characteristics of the material used for the truss, namely the Young's Modulus (E), which is the stiffness of the material when a force is applied lengthwise, the Yield Strength (Y), which is the stress at which 0.2% of the unstress length is added on in plastic deformation, the outer (OD) and inner (ID) dimensions of the tube material, which is then used to calculate the cross-sectional area (A, using the equation $A = OD^2 - ID^2$) alongside the Moment of Inertia (I, using the equation $I = \frac{1}{12}(OD^4 - ID^4)$). The Moment of Inertia describes the ability of each member to resist angular acceleration, and is used in conjunction with A in the equation $Rg = \sqrt{I/A}$, where Rg is the Radius of Gyration, which is the radial distance to a point that, if given the same mass as the member, would have the same Moment of Inertia as it. The Radius of Gyration is then used with Kt, a value representing the end conditions of the truss, or how the truss is supported in order to determine the Transition Length (Lt) used to convert Johnson's critical load to Euler's critical load. In our case, each truss used a hinge support at each end, giving a Kt value of 1. The equation was found by the following steps, where Sr = Slenderness Ratio, and Scr = Critical Slenderness ratio:

$Sr = \frac{Kt*L}{Rg}$, $L = \frac{Sr*Rg}{Kt}$

$Scr = \sqrt{\frac{2\pi*E}{Y}}$

Because Johnson's Critical Load, and Euler's Critical load use the Critical Slenderness Ratio, Sr=Scr in the above equation, therefore:

$L = Lt = \frac{\sqrt{\frac{2\pi*E}{Y}}*Rg}{Kt}$

Finally, Joint Reduction (Jr) must be entered, which is used in the empirical analysis of our modeler. This value represents the length of each member of the truss that is supported by the joint, which is taken into account for the empirical analysis, but not the theoretical analysis.

Once all of the above important material data is found, calculated, and entered, the next step in using our Truss Modeler is to enter the Member Lengths and Forces that were output by Kleinman's Truss Analyzer. Our modeler then uses the member length the user inputs as the Theoretical Unsupported length (Lut), as the theoretical section of the modeler assumes perfect, infinitesimally small joints for the members to connect to. Each unsupported length is used in the formula $PcrEuler_T = \frac{\pi^2 EI}{(KtL_{Ut})^2}$ to determine the Euler critical load and then the formula $PcrJohn_T = (\frac{Y-1}{E(\frac{Y}{2\pi})})^2 * (\frac{Kt*L_{ut}}{Rg})^2 * A$ to find the Johnson critical load. The two loads are compared for each member and the modeler chooses the lesser of the two to be the Theoretical Critical Load. The smallest value of the Theoretical Critical Load is then chosen as the Failure load, as when that load is reached a member fails on the truss, leading it to become unsafe. The sum of every inputted member length is entered to find the total length of material used. By dividing the theoretical failure load by the total material length, the modeler then finds the Strength to Length Ratio, a measure of how efficient the truss design is using the given material.

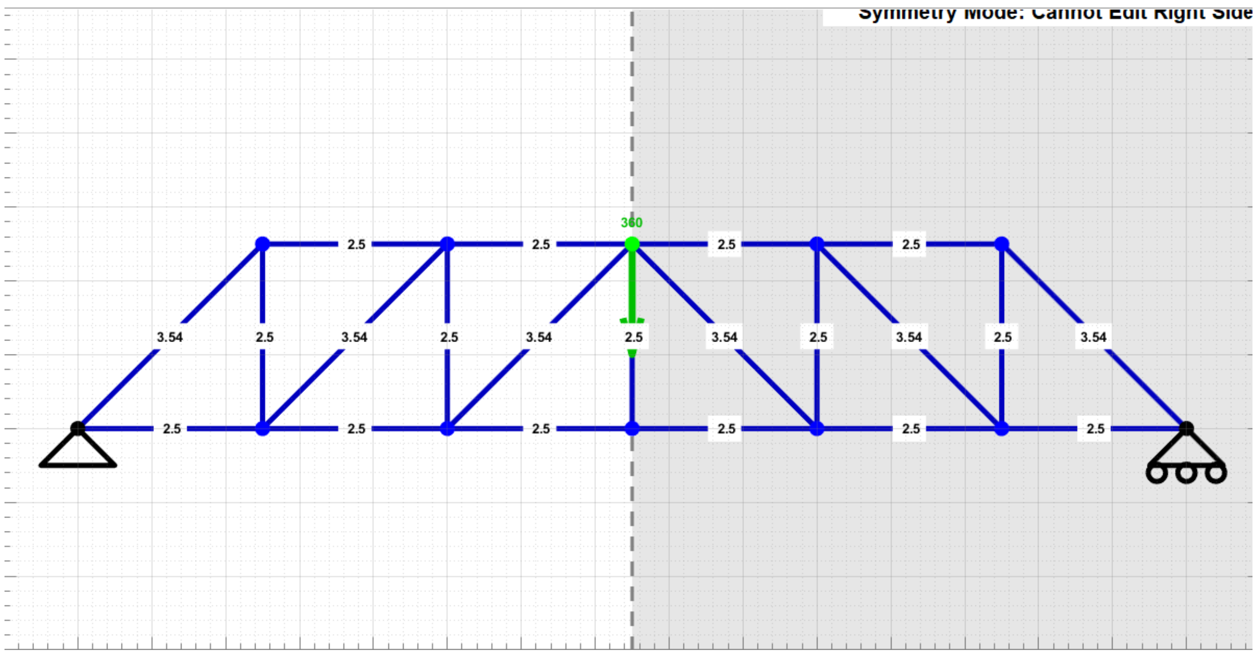
To find the empirical failure load of the given truss, the modeler first uses the equation $L_{uP} = L_{uT} - Jr$, where Lup is the empirical unsupported length. This takes into account any part of the member that is supported by the joints. The modeler then uses identical equations as in the theoretical section to calculate the empirical Euler and Johnson critical loads, except using Lup instead of Lut. As in the theoretical section, the lowest of the two values per member is selected to be the critical load for that member, and then the minimum value out of every critical load is chosen as the empirical failure load, which is then divided by the total material length to find the empirical Strength to Length ratio. The modeler then does some error analysis, using the equation $\%Error \; = \; \frac{\left|FailureLoad_T - FailureLoad_P\right|}{FailureLoad_t} * \; 100$ to determine the difference support from the joins would make on the design.

Discussion:

In order to not only develop an operational truss, but one which meets the suggested requirements, we undergo a multi-step process in order to design a completed model in the truss analyzer. For our first step we browse a number of truss designs, and select one which we wish to replicate. We then sketch out the general shape and length of each member, and assign a basic length unit to each. Following this we compare the drawn lengths to the lengths usable in the truss modeler. Due to equipment restrictions, we then adjust the lengths of the members such that the newer drawing possesses measurements as close to the original as possible while still maintaining the general shape of the chosen truss design. We then map out all the member loads, and the cost/material efficiency, and the failing members. From there we then tweak the lengths of the members, adjusting the shape of the truss such that the failing members receive less stress, without undergoing major changes. We keep repeating this full process, keeping an eye on material usage, while attempting to even out the loads as best possible.

Chris's Truss:

The model for this truss design was based around the multiple king post truss. There were a number of issues with designing the initial lengths as the original design ideas were to incorporate 2 or 4 sections per side, but due to each side of the truss modeler only having 7.5 in length led to only three sections. The truss was originally meant to be a different truss, with a slight curve at the top, however I was unable to accurately replicate this design, and instead used a flat top. A 360 Newton force was applied to the top of the truss for testing stresses.



This is the final truss design, with member lengths.

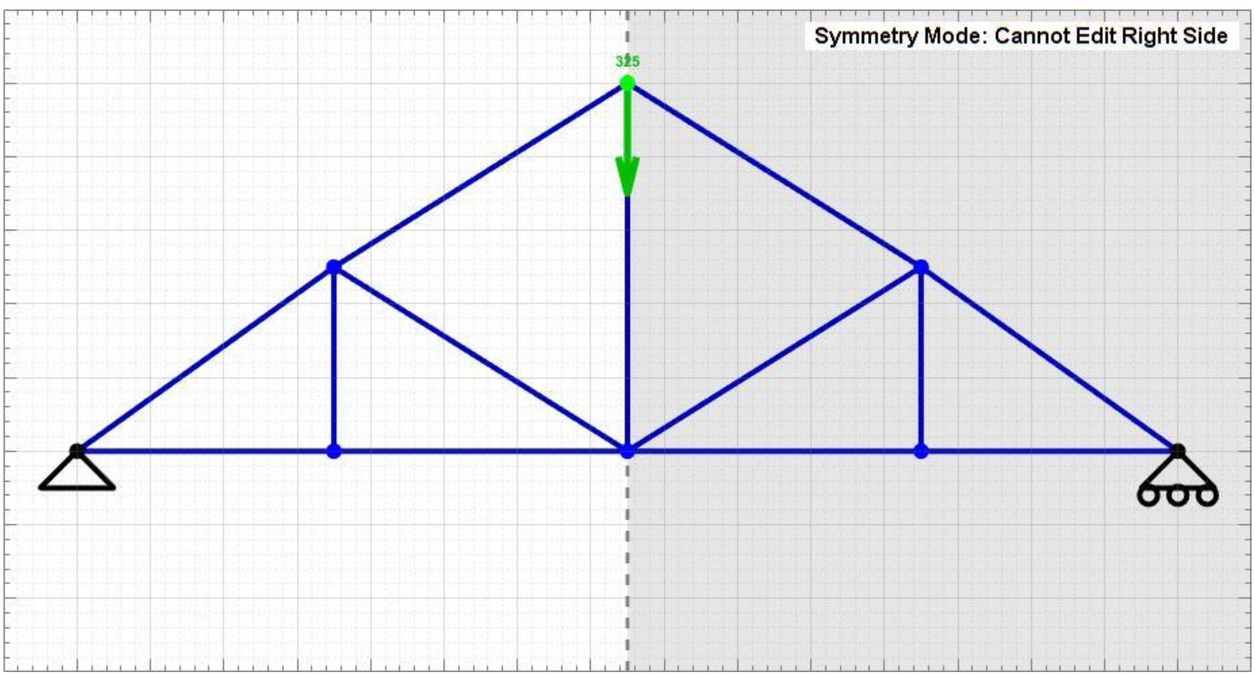
TrussAnalyzer Output			Theoretical					Physical (empirical)				
M#	ML	MF	Lu_T	PcrEuler_T	PcrJohn_T	Pcr_T	Lf_T	Lu_P	PcrEuler_P	PcrJohn_P	Pcr_P	Lf_P
1	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
2	3.54	-254.5	3.54	163.40796	158.33488	163.408	0.642075	2.54	317.40393	268.01568	268.0157	1.053107
3	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
4	3.54	-254.5	3.54	163.40796	158.33488	163.408	0.642075	2.54	317.40393	268.01568	268.0157	1.053107
5	2.5	360	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
6	3.54	-254.6	3.54	163.40796	158.33488	163.408	0.641822	2.54	317.40393	268.01568	268.0157	1.052693
7	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
8	2.5	540	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
9	3.54	-254.6	3.54	163.40796	158.33488	163.408	0.641822	2.54	317.40393	268.01568	268.0157	1.052693
10	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
11	2.5	0	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
12	2.5	540	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
13	2.5	-360	2.5	327.64211	271.65247	271.6525	0.75459	1.5	910.11698	343.81089	343.8109	0.95503
14	3.54	-254.6	3.54	163.40796	158.33488	163.408	0.641822	2.54	317.40393	268.01568	268.0157	1.052693
15	2.5	-360	2.5	327.64211	271.65247	271.6525	0.75459	1.5	910.11698	343.81089	343.8109	0.95503
16	2.5	-180	2.5	327.64211	271.65247	271.6525	1.50918	1.5	910.11698	343.81089	343.8109	1.91006
17	2.5	360	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
18	3.54	-254.6	3.54	163.40796	158.33488	163.408	0.641822	2.54	317.40393	268.01568	268.0157	1.052693
19	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
20	2.5	180	2.5	327.64211	271.65247	271.6525		1.5	910.11698	343.81089	343.8109	
21	2.5	-180	2.5	327.64211	271.65247	271.6525	1.50918	1.5	910.11698	343.81089	343.8109	1.91006

These are the results of the final truss design.

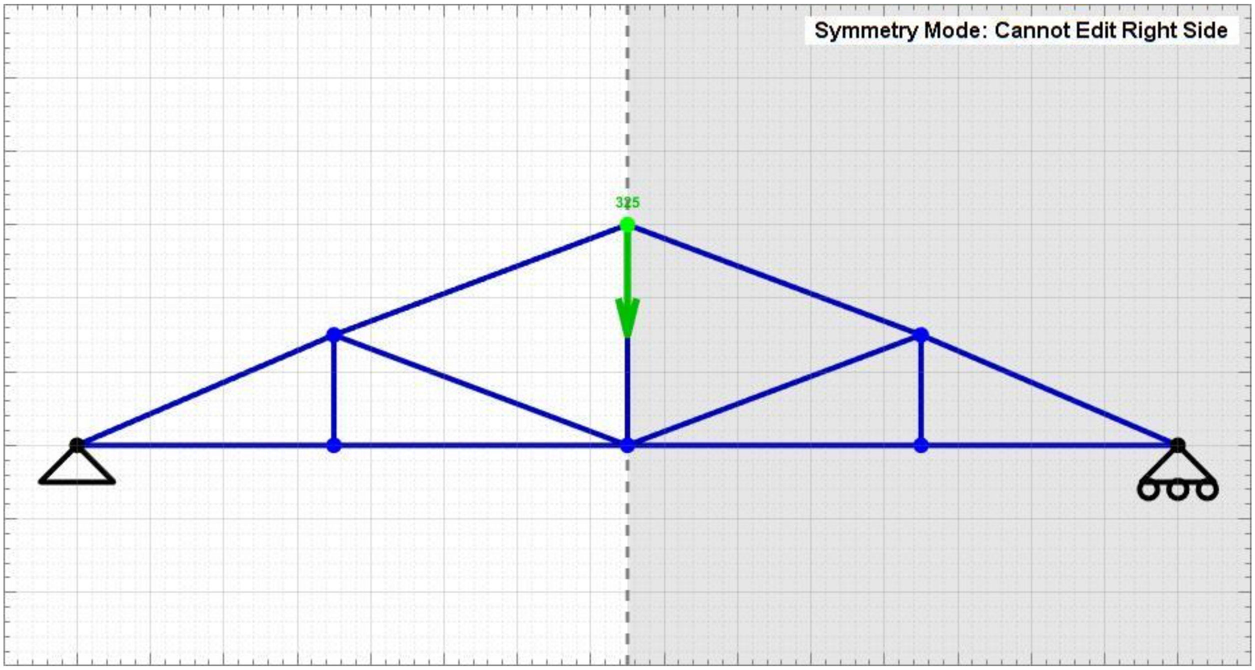
Joe’s Truss:

Joe's design was modeled after the Waddel "A" Truss. This design was chosen due to the few members allowing for the design to very easily meet the objective of remaining under 60". The initial design took a 5" height with supporting members at the midpoint of the outer members, but the strength to length ratio of this design was relatively low, so Joe refined the design by altering the height of the truss and inserting the values into the truss modeler, and he found that up to the point where the empirical analysis gave a length of 0 (due to the members being the length supported by the joints) that as he lowered the height of his truss the strength to length ratio increased significantly, from an initial 2.44 to 4.19.

Pictured below is the initial truss design:



Pictured below is the final design:



Pictured below is the modeler analysis of Joe's final design:

TrussAnalyzer Output			Theoretical				Physical (empirical)			
M#	ML	MF	Lu_T	PcrEuler_T	PcrJohn_T	Pcr_T	Lu_P	PcrEuler_P	PcrJohn_P	Pcr_P
1	3.5	379.233	3.50	167.16	163.09	167.16	2.50	327.64	271.65	271.65
2	3.807887	-412.461	3.81	141.22	122.44	141.22	2.81	259.73	242.17	242.17
3	3.5	379.233	3.50	167.16	163.09	167.16	2.50	327.64	271.65	271.65
4	3.807887	-412.461	3.81	141.22	122.44	141.22	2.81	259.73	242.17	242.17
5	4	379.1691	4.00	127.98	95.34	127.98	3.00	227.53	222.04	222.04
6	3	-20.3125	3.00	227.53	221.81	221.81	2.00	511.94	312.24	312.24
7	4.272002	28.92736	4.27	112.21	54.69	112.21	3.27	191.27	191.27	191.27
8	4	379.1691	4.00	127.98	95.34	127.98	3.00	227.53	222.04	222.04
9	4.272002	28.92736	4.27	112.21	54.69	112.21	3.27	191.27	191.27	191.27
10	1.5	0.013112	1.50	910.12	343.75	343.75	0.50	8191.05	379.89	379.89
11	4.272002	-433.874	4.27	112.21	54.69	112.21	3.27	191.27	191.27	191.27
12	4.272002	-433.874	4.27	112.21	54.69	112.21	3.27	191.27	191.27	191.27
13	1.5	0.013112	1.50	910.12	343.75	343.75	0.50	8191.05	379.89	379.89

Theoretical	
FailureLoad_T (lbs)	112.21
Total Brass Length (in)	45.703782
Strength to Length Ratio	2.46

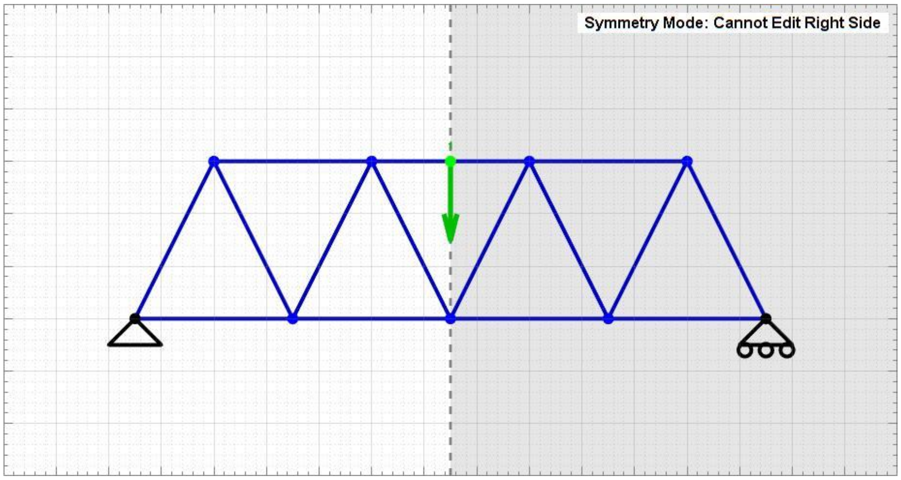
Physical (empirical)	
FailureLoad_P (lbs)	191.27
Total Brass Length (in)	45.70
Strength to Length Ratio	4.19

Error Analysis	
%Failure T/P	70.47%

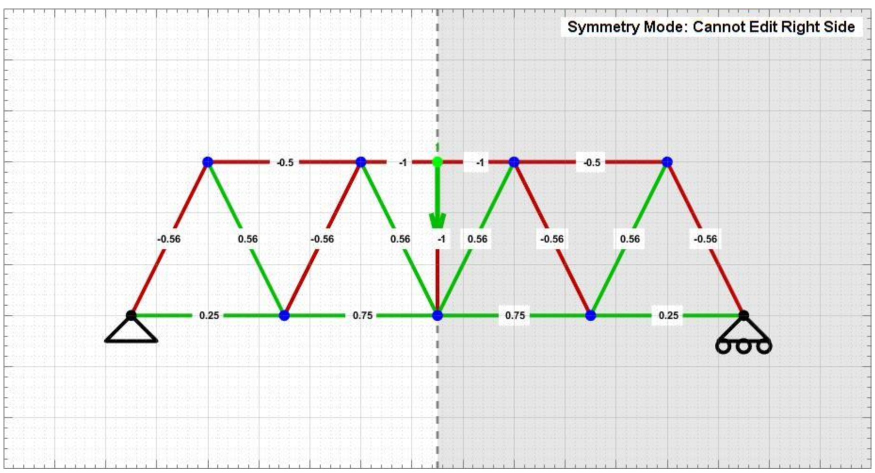
Angel’s Truss:

In Angel’s design, we can see a very simple truss design modeled after the classic Warren truss. The first iteration of the design was not working out because the Truss condition was not being met in the truss program, so the program would not show the forces and the failing members. Angel fixed this problem by adding a middle member to the truss design, this then completed the truss condition and allowed the program to display the necessary information to fill out the modeler. Angel’s design went through some other changes prior to being photographed, originally the truss was going to be much taller and a bit longer as well, but Angel cut down on these dimensions in order to fulfill the project requirements. The most difficult part of designing this truss for Angel was to make it fit under the specified length. Angel’s trusses kept on being too long for the project constraints, causing him to think that he was adding too many members, when in reality the problem was that his truss was just too long and tall. Once Angel figured out that the problem was the truss’s overall height and length, he was able to make his Warren-esque truss design work and fit under the project’s constraints.

Pictured here is the design which Angel thought would be the final design, but was not working as a truss in the program:



Pictured here is the final design, which acted as a truss in the program and allowed Angel to fill out the modeler successfully:



Angel's results from the truss analyzer program and the equations that were discussed in the introduction were as follows (these values were found by applying a 325 pound load at the apex of the truss):

TrussAnalyzer Output			Theoretical				Physical (empirical)			
M#	ML	MF	Lu_T	PcrEuler_T	PcrJohn_T	Pcr_T	Lu_P	PcrEuler_P	PcrJohn_P	Pcr_P
1	3.35	-181.7	3.35	183	182	183	2.35	371	285	285
2	3	81.26	3	228	222	222	2	513	312	312
3	3.35	-181.7	3.35	183	182	183	2.35	371	285	285
4	3	81.26	3	228	222	222	2	513	312	312
5	3	-325	3	228	222	222	2	513	312	312
6	3.35	181.7	3.35	183	182	183	2.35	371	285	285
7	3	243.8	3	228	222	222	2	513	312	312
8	3.35	181.7	3.35	183	182	183	2.35	371	285	285
9	3	243.8	3	228	222	222	2	513	312	312
10	1.5	-325	1.5	911	344	344	0.5	8203	380	380
11	1.5	-325	1.5	911	344	344	0.5	8203	380	380
12	3	-162.5	3	228	222	222	2	513	312	312
13	3.35	-181.7	3.35	183	182	183	2.35	371	285	285
14	3.35	181.7	3.35	183	182	183	2.35	371	285	285
15	3	-162.5	3	228	222	222	2	513	312	285
16	3.35	-181.7	3.35	183	182	183	2.35	371	285	285
17	3.35	181.7	3.35	183	182	183	2.35	371	285	285

Angel’s truss ended up being 50.83 inches in total length, with a theoretical strength to length ratio of 3.6, and an empirical strength to length ratio of 5.6.

Conclusion:

Seeking to fulfill our initial design conditions we each designed multiple trusses. Through a multi-step iterative design process, we attempted to improve the capabilities of our trusses and gain a better understanding of how they worked. In order to properly analyze and subsequently understand the effects resulting from the changes we made, we created and utilized an excel sheet to compare the stresses each truss iteration faced and how they changed with each design decision. In the end this was a very educational process, teaching a lot about the design process. We learned that in general, a simpler design is superior. We also noted that having smaller members usually resulted in a stronger design. There was a great deal of difficulty introduced into this assignment due to constraints on the design size. The truss modeler program required all bridges be a specific length which limited our design space, and prohibited clean division of the distance as there was a limited number of lines between the two pre-designated end points. While in the real world there often would be very specific measurements a design would have to fit within, being unable to adjust them or easily model fit lengths along the bottom of the truss limited how we could experiment.